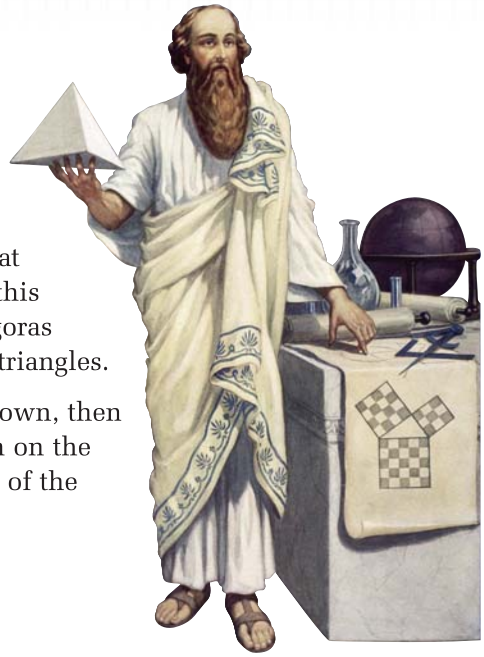
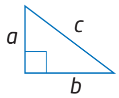
8.1 Apply the Pythagorean Theorem

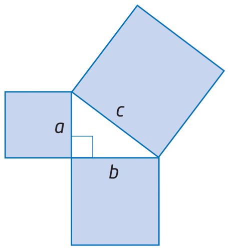


The Pythagorean theorem is named after the Greek philosopher and mathematician Pythagoras (580–500 B.C.E.). Although ancient texts indicate that different civilizations understood this property of right triangles, Pythagoras proved that it applies to all right triangles.

If a right triangle is labelled as shown, then the area of the large square drawn on the **hypotenuse** is c^2, while the areas of the other two squares are a^2 and b^2.

hypotenuse
- the longest side of a right triangle
- the side opposite the 90° angle





According to the Pythagorean relationship, the area of the square drawn on the hypotenuse is equal to the sum of the areas of the squares drawn on the other two sides.

Therefore, the algebraic model for the Pythagorean relationship is $c^2 = a^2 + b^2$. This is known as the **Pythagorean theorem**.

Pythagorean theorem
- in a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the two shorter sides

Investigate

How can you illustrate the Pythagorean theorem?

Tools
- grid paper
- ruler

Method 1: Use Pencil and Paper

1. Construct any right triangle. Label the sides of your triangle using three different letters.

2. Measure the length of each side of your triangle. Indicate these measures on your diagram.

3. a) Calculate the area of the square on the hypotenuse.

b) Calculate the sum of the areas of the squares on the two shorter sides.

c) Write the Pythagorean theorem using your side labels.

4. **a)** Calculate the square root of your answer to step 3b).
   b) Compare this value to the length of the hypotenuse.

5. Construct any non-right triangle. Does the Pythagorean relationship still hold? Does the relationship from step 4, part b), still hold?

6. **Reflect** Explain how this activity illustrates the Pythagorean theorem.

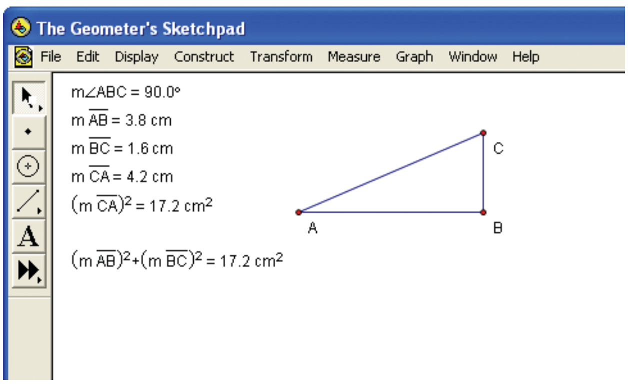
Method 2: Use *The Geometer's Sketchpad*®

1. From the **Edit** menu, choose **Preferences.** Click on the **Units** tab. Set the precision to tenths for all three boxes. Click on the **Text** tab and check **For All New Points**. Click on **OK**.

2. Use the **Straightedge Tool** to create any △ABC.

3. **a)** To measure ∠ABC, select vertices A, B, and C, in that order. From the **Measure** menu, choose **Angle**.
   b) To measure the length of AB, select line segment AB. From the **Measure** menu, choose **Length**. Repeat for line segments BC and CA.

4. **a)** Drag a vertex of the triangle until ∠ABC measures 90°.
   b) Select the measure $\mathbf{m\overline{CA}}$. From the **Measure** menu, choose **Calculate**. Enter $\mathbf{m\overline{CA}}$^2, by selecting $\mathbf{m\overline{CA}}$ from the **Values** drop-down menu on the calculator.
   c) Select $\mathbf{m\overline{AB}}$ and $\mathbf{m\overline{BC}}$. From the **Measure** menu, choose **Calculate**. Enter $\mathbf{m\overline{AB}}$^2 + $\mathbf{m\overline{BC}}$^2.



Tools
- computers
- *The Geometer's Sketchpad*® software

Go to www.mcgrawhill.ca/links/principles9 and follow the links to an interactive proof of the Pythagorean theorem.

Did You Know?

To create a right angle for measuring land or building pyramids, the ancient Egyptians tied 12 equally spaced knots in a rope. They then tied the rope into a loop and stretched it to form a triangle with a knot at each vertex. The only way this works is in the ratio 3:4:5, resulting in a right triangle.

5. a) Select $(m\overline{AB}^2)$ and $(m\overline{BC}^2)$. From the **Measure** menu, choose **Calculate**. Evaluate $\sqrt{(m\overline{AB})^2 + (m\overline{BC})^2}$ by choosing **sqrt** from the **Functions** pull-down menu on the calculator.

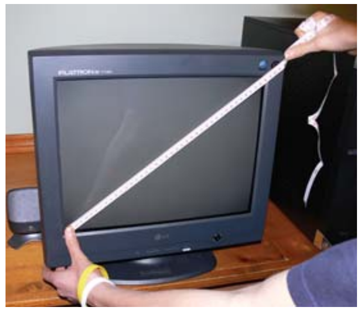
b) Compare this value to the length of side CA.

6. Drag a vertex of the triangle so that the measure of ∠ABC is no longer 90°. Does the Pythagorean relationship still hold? Does the relationship from step 5b) still hold?

7. **Reflect** Explain how this activity illustrates the Pythagorean theorem.

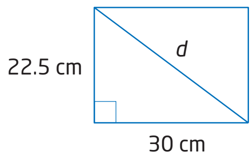
Example 1 Find the Hypotenuse

The advertised size of a computer or television screen is actually the length of the diagonal of the screen. A computer screen measures 30 cm by 22.5 cm. Determine the length of its diagonal.



Solution

In the diagram, the diagonal, d, is the hypotenuse.



Apply the Pythagorean theorem.

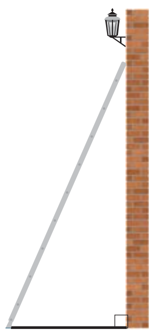
$$d^2 = 30^2 + 22.5^2$$
$$d^2 = 900 + 506.25$$
$$d^2 = 1406.25$$
$$\sqrt{d^2} = \sqrt{1406.25}$$
$$d = 37.5$$

Only the positive square root needs to be used because *d* is a length.

The length of the diagonal of the computer screen is 37.5 cm.

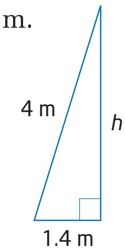
Example 2 Find One of the Shorter Sides

Jenna is changing a light bulb. She rests a 4-m ladder against a vertical wall so that its base is 1.4 m from the wall. How high up the wall does the top of the ladder reach? Round your answer to the nearest tenth of a metre.



Solution

In this case, the ladder is the hypotenuse, with a length of 4 m. The unknown side length is h.



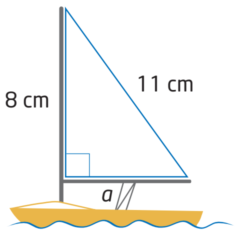
Apply the Pythagorean theorem.

$$\begin{aligned}
4^2 &= 1.4^2 + h^2 \\
16 &= 1.96 + h^2 \\
16 - 1.96 &= 1.96 - 1.96 + h^2 && \textbf{Subtract 1.96 from both sides.} \\
14.04 &= h^2 \\
\sqrt{14.04} &= \sqrt{h^2} && \textbf{Take the square root of both sides.} \\
3.7 &\doteq h
\end{aligned}$$

The ladder reaches 3.7 m up the wall, to the nearest tenth of a metre.

Example 3 Calculate the Area of a Right Triangle

Calculate the area of the triangular sail on the toy sailboat.



Solution

The formula for the area of a triangle is $A = \frac{1}{2}bh$.

The base, b, and the height, h, must be perpendicular to each other. For a right triangle, the base and the height are the lengths of the two shorter sides.

First, use the Pythagorean theorem to find the length of the unknown side, a.

$$\begin{aligned}
11^2 &= a^2 + 8^2 \\
121 &= a^2 + 64 \\
121 - 64 &= a^2 + 64 - 64 && \textbf{Subtract 64 from both sides.} \\
57 &= a^2 \\
\sqrt{57} &= \sqrt{a^2} && \textbf{Take the square root of both sides.} \\
a &\doteq 7.5
\end{aligned}$$

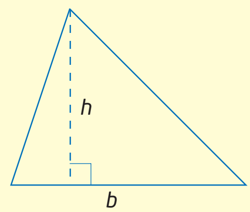
The length of side a is approximately 7.5 cm.

Now, apply the formula for the area of a triangle.

$$\begin{aligned}
A &= \frac{1}{2}bh \\
&= \frac{1}{2}(8)(7.5) \\
&= 30
\end{aligned}$$

The area of the sail is approximately 30 cm^2.

I can write the area formula for a triangle in different ways: $A = \frac{1}{2}bh$, $A = \frac{bh}{2}$, and $A = 0.5bh$.

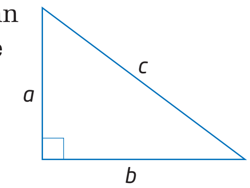


Literacy Connections

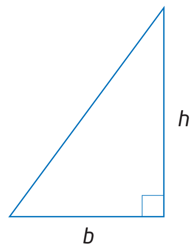
The perpendicular sides of a right triangle are called the legs of the triangle.

Key Concepts

- The longest side of a right triangle is the hypotenuse.
- The Pythagorean theorem states that the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the two shorter sides.
- An algebraic model representing the Pythagorean theorem is $c^2 = a^2 + b^2$, where c represents the length of the hypotenuse and a and b represent the lengths of the two shorter sides.

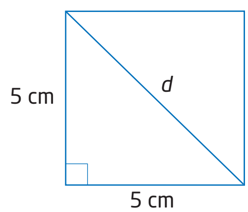


- You can use the Pythagorean theorem to calculate the length of an unknown side of a right triangle.
- You can calculate the area of a right triangle by using the formula $A = \frac{1}{2}bh$, with the lengths of the two shorter sides as the base, b, and the height, h. If one of these dimensions is unknown and you know the hypotenuse, apply the Pythagorean theorem to calculate the length of the unknown side. Then, use the area formula.

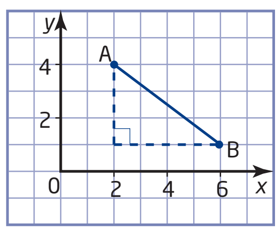


Communicate Your Understanding

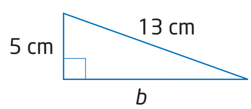
C1 Describe how you can use the Pythagorean theorem to determine the length of the diagonal of the square.



C2 Describe how you can use the Pythagorean theorem to determine the distance between two points on a grid.



C3 Describe how you would find the area of a right triangle if you knew the lengths of the hypotenuse and one of the other two sides.

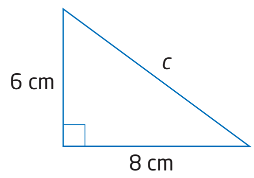


Practise

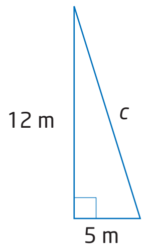
For help with question 1, see Example 1.

1. Calculate the length of the hypotenuse in each triangle. Round your answers to the nearest tenth of a unit, when necessary.

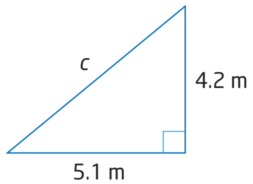
a)



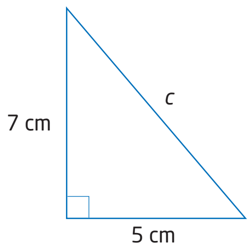
b)



c)



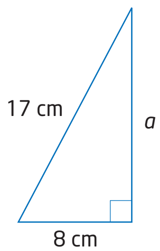
d)



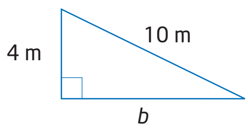
For help with question 2, see Example 2.

2. Calculate the length of the unknown side in each triangle. Round your answers to the nearest tenth of a unit, when necessary.

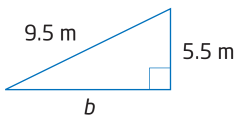
a)



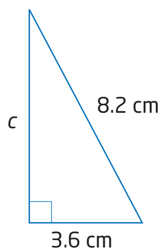
b)



c)



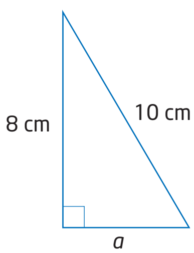
d)



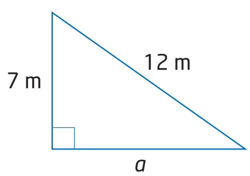
For help with question 3, see Example 3.

3. Determine the area of each right triangle. Round your answers to the nearest tenth of a square unit, when necessary.

a)



b)



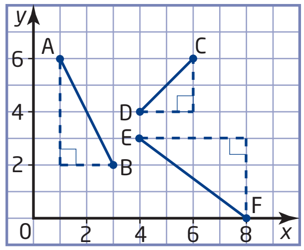
Connect and Apply

4. Calculate the length of each line segment. Round answers to the nearest tenth of a unit, when necessary.

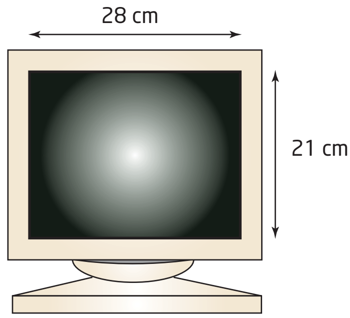
a) AB

b) CD

c) EF



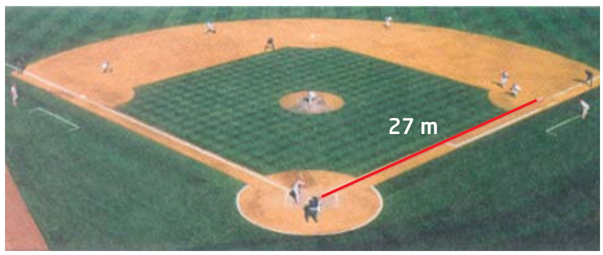
5. What is the length of the diagonal of a computer screen that measures 28 cm by 21 cm?



Did You Know?

Baseball was formally introduced as a medal sport at the 1992 Summer Olympics in Barcelona, Spain. Canada made its first appearance in this event in the 2004 Summer Olympics.

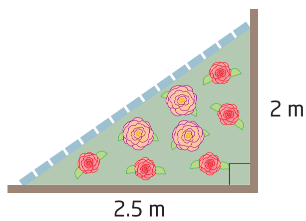
6. A baseball diamond is a square with sides that measure about 27 m. How far does the second-base player have to throw the ball to get a runner out at home plate? Round your answer to the nearest metre.



7. A square courtyard has diagonal paths that are each 42 m long. What is the perimeter of the courtyard, to the nearest metre?

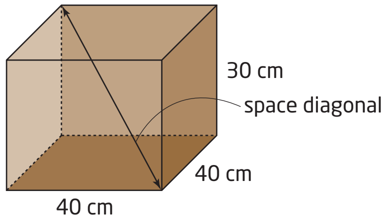
8. Brook is flying a kite while standing 50 m from the base of a tree at the park. Her kite is directly above the 10-m tree and the 125-m string is fully extended. Approximately how far above the tree is her kite flying?

9. **Chapter Problem** Emily has designed a triangular flower bed for the corner of her client's rectangular lot. The bed is fenced on two sides and Emily will use border stones for the third side. The bed measures 2 m and 2.5 m along the fenced sides. How many border stones, 30 cm in length, will Emily need to edge the flower bed?



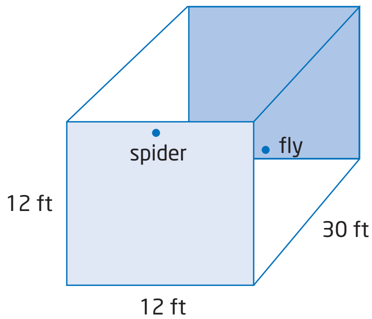
Extend

10. A cardboard box measures 40 cm by 40 cm by 30 cm. Calculate the length of the space diagonal, to the nearest centimetre.



11. The Spider and the Fly Problem is a classic puzzle that originally appeared in an English newspaper in 1903. It was posed by H.E. Dudeney. In a rectangular room with dimensions 30 ft by 12 ft by 12 ft, a spider is located in the middle of one 12 ft by 12 ft wall, 1 ft away from the ceiling. A fly is in the middle of the *opposite* wall 1 ft away from the floor. If the fly does not move, what is the shortest distance that the spider can crawl along the walls, ceiling, and floor to capture the fly?

Hint: Using a net of the room will help you get the answer, which is less than 42 ft!

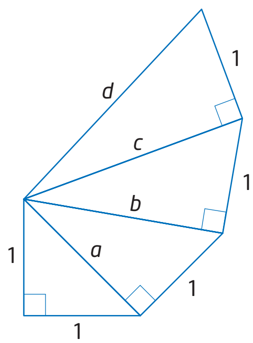


12. A spiral is formed with right triangles, as shown in the diagram.

a) Calculate the length of the hypotenuse of each triangle, leaving your answers in square root form. Describe the pattern that results.

b) Calculate the area of the spiral shown.

c) Describe how the expression for the area would change if the pattern continued.



13. Math Contest

a) The set of whole numbers (5, 12, 13) is called a *Pythagorean triple*. Explain why this name is appropriate.

b) The smallest Pythagorean triple is (3, 4, 5). Investigate whether multiples of a Pythagorean triple make Pythagorean triples.

c) Substitute values for m and n to investigate whether triples of the form $(m^2 - n^2, 2mn, m^2 + n^2)$ are Pythagorean triples.

d) What are the restrictions on the values of m and n in part c)?